Optimum Design of Open Reinforced Concrete Circular Cylindrical Tanks Rest on Ground

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Abstract

A computer program has been generated to calculate the optimum dimensions and the amount of reinforcements for open reinforced concrete circular cylindrical tanks rest on ground. The design is based on limit state method for both ultimate and serviceability limit states in accordance with the British Standards B.S. 8110 and B.S. 5337. The cost of concrete, steel, and formwork are considered. The procedure is based on the interior penalty method to find the optimum solution for the non-linear programming problem. The tank consists of cylindrical wall and circular base and the joint between them was considered as partially fixed. The design variables consist of tank geometric variables in addition to steel content in seven positions. The effect of the design capacity of the tank, bearing capacity of the soil, unit price of steel and concrete, and finally unit cost of formwork was studied. It is found that the reduction of the bearing capacity of the soil linearly increases the cost of the tank. The increase of concrete and steel unit costs leads to increasing the tank height while the increase of formwork unit cost enhances the tank diameter, to reach the optimal design.

التصميم الأمثل للخزانات الأسطوانية الخرسانية المسلحة المفتوحة المسندة على الأرض

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الخلاصة

تم استخدام تقنيات رياضية برمجية لتصميم خزان أسطواني مسلح مفتوح مستند على الأرض بأقل كلفة وباستخدام الطريقة المسماة دالة الجزاء لإيجاد الحل الأمثل للمسالة غير الخطية.

شملت المتغيرات التصميمية للمسألة إبعاد الخزان بالإضافة إلى سبعة أنواع مختلفة من حديد التسليح وإبعاد الخزان تتكون من القطر الداخلي للخزان، الارتفاع الكلي، سمك الجدار و سمك الأرضية. بينما يشمل حديد التسليح الحديد في كل وجه من الجدار بالاتجاهين الطولي والمحيطي كذلك الحديد في أسفل وأعلى القاعدة أما المفصل بين الجدار والقاعدة فقد تم اعتباره ثابتكجزئيا.

درس تأثير عدة عوامل شملت سعة الخزان، قابلية التحميل الأمينة للتربة، نوع الخرسانة المستخدمة وكذلك حديد التسليح المستخدم، وأخيرا سعر الخرسانة، الحديد، وإعمال القالب. المقيدات التي ثبتت في المسالة هي:

1- متطلبات التحمل استنادا إلى المواصفات البريطانية 5337 و 8110
2- متطلبات الخدمة استنادا إلى المواصفات البريطانية 5337 و 8110

دالة الهدف تمثل كلفة الخزان (الجدار الأسطواني والقاعدة الدائرية) وتشتمل على كلفة المواد إضافة إلى أجور العمل.

تم حل عدة أمثلة لتوضيح تأثير المتغيرات التصميمية على الحل الأمثل للمسالة، وكذلك تفسير تأثير العوامل المذكورة أعلاه على التصميم الأمثل حيث وضحت هذه التأثيرات من خلال مجموعة من الجداول والمخططات التي يمكن استخدامها لإيجاد التصميم الأمثل لأي مجموعة من هذه المعاملات. تبين من خلال النتائج المستحصلة أن الخزان الأمثل هو الخزان القصير وكذلك فان تقليل قابلية التحمل الأمينة للتربة أدت إلى زيادة خطية لكلفة الخزان. وأخيرا فان الزيادة في سعر الخرسانة والحديد أدت إلى زيادة ارتفاع الخزان بينما أدت زيادة سعر القالب إلى كبر قطر الخزان للوصول إلى التصميم الأمثل.

INTRODUCTION

Tanks are made from different materials (single or composite) in many shapes and sizes for different purposes. In general, reinforced concrete water tanks can be classified into tanks resting on ground, elevated tanks, and underground tanks. A familiar type of tanks is the open circular cylindrical reinforced concrete tank resting on ground. This type is widely used in different branches of civil engineering, it consists of cylindrical wall and circular base and the joint between them was considered as partially fixed [1]. It has less number of joints, and the placement of steel reinforcement and manufacturing of formwork are easier than other types. The cost of tank consists of cost of the wall and cost of the base.

The cost optimum design of various reinforced concrete structures is receiving more and more attention from the researchers. Traum [2] in 1962 presented a method for the economical design of slabs. Chou [3] in 1977 discussed the optimum design of reinforced concrete T-beam sections. Kirsch [4] in 1984 developed the multilevel approach in the optimum structural design for buildings. Azmy and Eid [5] in 1999 gave an optimization procedure for the shear design of rectangular beams.

This paper deals with the optimum design of open reinforced concrete circular cylindrical tanks in accordance with British standards B.S. 8110 [6] and B.S. 5337 [7] using the limit state method for both ultimate and serviceability requirements. The effects of many parameters, including the design capacity of the tank, the bearing capacity of the soil, the unit cost of steel, concrete, and formwork are investigated.

Method of Analysis

The tank consists of cylindrical wall and circular base connected together at the junction joint that is considered partially fixed. Since the present problem is axi-symmetrical problem for both geometry and loading then the analysis is considered for unit length of wall perimeter at the mean diameter and a part of the base connected to it. The analysis of tank is carried out for two load cases; the first is when the tank is full and the second when it is empty. The deflected shape of the wall is shown in Fig. (1), only a rotation is allowed at the common joint while no translation for both wall and base occurs.

The compatibility and equilibrium requirements are [8]:

1-The rotations of wall and base are the same at the junction joint.

2-The moments in wall and base are the same at the junction joint.

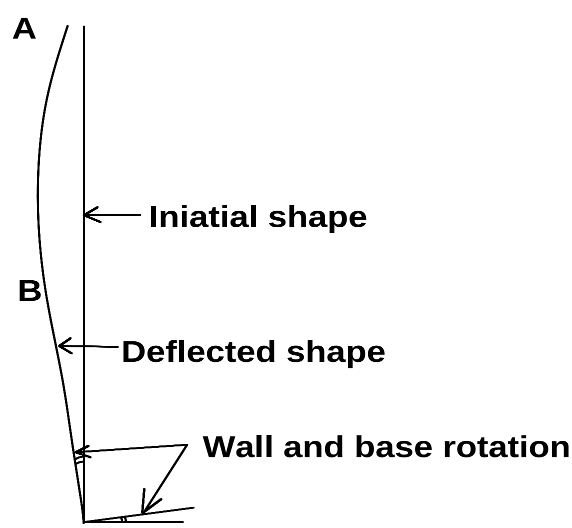

Fig.1 The wall deformation of partilly fixed joint

The analytical method [1] is used for the analysis of wall combined with the differential equation of base [8] while the limit state method is used in the design [6, 7]. The forces that are considered in the analysis and design include bending moments, shear forces and direct tension for ultimate limit state and a check is made for crack widths and steel stress in serviceability limit state.

For the first load case, when the tank is full with water, the hydrostatic pressure is resisted by two actions, hoop tension and bending (cantilever action) of wall. Also at any section, at depth x below the water surface, the deformation y_r due to hoop tension will be equal to the displacement y_c due to cantilever action. The sum of loads transferred due to both cantilever action and hoop tension is equal to intensity of water pressure at that section [1]. Hence

$$p_c + p_r = p_x = \gamma_w . x \text{ ---------------------------- (1)}$$

where γ_w : weight of water per unit volume;

For compatibility $y_c = y_r = y$ ---------- (2)

where p_c : load transferred due to cantilever action (pressure per unit area) or horizontal shear i.e.

$$p_c = EI\left(\frac{d^4 y}{dx^4}\right) \text{ ------------------------------- (3)}$$

p_r : load carried by ring tension (pressure per unit area) i.e.

$$p_r = \frac{4 \times T_1 \times E \times y_r}{D^2} \text{ -------------------------- (4)}$$

p_x : hydrostatic pressure and equals to $\gamma_w . x$

y : change in radius at depth x.

D : inner diameter of the tank;

E : modulus of elasticity of concrete;

T_1 : wall thickness;

Substituting p_r and p_c from Esq. 3 and 4 into Eq. 1 results in:

$$EI\frac{d^4 y}{dx^4} + \frac{4T_1 \times E}{D^2} y = \gamma_w . x \text{ ------------- (5)}$$

The applied forces on wall at junction joint (bending moment and shear force) are determined from the following equations [8,9]:

$$M_f = -4\alpha^2 EI \times A_1(\cos\alpha.H \sinh\alpha.H - e^{-\alpha.H}\sin\alpha.H) + A_2(4\alpha^2 EI \sin\alpha.H \sinh\alpha.H) \text{ -----------(6)}$$

$$V_u = -4EI\alpha^3\{A_1(\cos\alpha H\cosh\alpha H - \sin\alpha H \sinh\alpha H - e^{-\alpha H}(\sin\alpha H + \cos\alpha H)) - A_2(\sin\alpha H\cosh\alpha H + \cos\alpha H \sinh\alpha H)\} \text{ ------------------(7)}$$

$$T = \frac{2ET_1}{D}\{\frac{\gamma_w . H^2}{8EI\alpha^4} + \frac{A_1}{\alpha}[\sin\alpha H(\sinh\alpha H - e^{-\alpha H}) - \cos\alpha H(\cosh\alpha H - e^{-\alpha H}) + 2] + \frac{A_2}{\alpha}(\sin\alpha H\cosh\alpha H + \cos\alpha H\sinh\alpha H - 1)\} \text{ --(8)}$$

where $\alpha^4 = \frac{T_1}{I \times D^2}$ ---------------------- (9)

H : height of water without free board (F.B);

I : moment of inertia of wall per unit length and equals to $[I = \frac{T_1^3}{12 \times (1-\mu^2)}]$

μ : Poisson's ratio for concrete = 0.17;

A_1 and A_2 are constants to be determined from the conditions that no translation occurs at the joint and the compatibility of rotations and equilibrium of moments at junction joint, also they are depending on the load case.

For this load case, there are three unknowns A_1, A_2, and M_f, thus three equations are needed. The first is obtained from the condition:

$y = 0$ at $x = H$ which results in:

$a_{11}A_1 + a_{12}A_2 = -b_1$ lead to

$A_1 = (b_1 - a_{12}A_2)/a_{11}$ ----------- (10)

The second equation is achieved from moment condition at the joint (Eq.6), i.e.

$M_f = a_{31}A_1 + a_{32}A_2$ --------------(11)

The third equation is also obtained from the condition at $x = H$ that the rotations (ϕ) are equal of the wall(ϕ_w) and the base (ϕ_A) [8,9], i.e.

$$\phi_w = \frac{dy}{dx} = a_{21}A_1 + a_{22}A_2 + b_2 \text{ ------------(12)}$$

$$\phi_A = \frac{1}{3D_s \times b_{av}}\sqrt{(M_f)^3/q} \text{ -------- (13)}$$

where D_s: flexural rigidity of base

$$D_s = \left[\frac{b_{av} \times T_2{}^3}{12(1-\mu^2)}\right] \times E$$

T_2: Base thickness.

b_{av}: average width of base sector that treated as a beam of length (L) i.e.

$$b_{av} = 1 - \frac{L}{D+T_1} \text{ -------------------------------(14)}$$

$$L = 2\sqrt{\frac{M_f}{q}} \text{ , and } q = \gamma_w.H$$

since $\phi = \phi_w = \phi_A$, then equating Eqs. 12 and 13 results in:

$$(a_{21}A_1 + a_{22}A_2 + b_2) \times (3D_s \times b_{av}\sqrt{q}) = (M_f)^{1.5} \text{ --(15)}$$

Substituting b_{av} and M_f values from Eq. 11 and Eq. 14 into Eq. 15 results in:

$$3D_s\sqrt{q}(C_1A_2 + C_2) \times \left(1 - \frac{2\sqrt{-(C_3A_2 + C_4)}}{(D+T_1)\sqrt{q}}\right)$$

$$= (-(C_3A_2 + C_4))^{1.5} \text{ --------------(16)}$$

Solving this equation for A_2 and then the other unknowns may be found while C_i and $a_{i,j}$ are constants that given in references [8,9].

For empty tank, no hydrostatic pressure exists and there is only reaction of ground on the base. This reaction results from the weight of the cylindrical wall only and distributed over the entire area of base as uniformly distributed load. The deflected shape of the tank is shown in Fig. (2), in which q_{ua} is the reaction of ground to base per unit area and equal to:

$$q_{ua} = \frac{4(D+T_1) \times H_t \times T_1 \times \gamma_c}{(D+2T_1)^2} \text{ ------- (17)}$$

where H_t: height of tank and equal to height of water (H) and free board distance (F.B);

γ_c: weight of concrete per unit volume;

The bending moment at the base center is found from the following equation [11]:

$$Mc = M_e - \frac{q_{ua} \times (3+\mu) \times (D+T_1)^2}{64} \text{ ---- (18)}$$

where M_e: the moment at junction joint for both base and wall which may be determined from solving the following equations where the constants $a_{i,j}$ are same for the first load case.

$$\begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & \dfrac{(D+T_1)}{2\times(1+\mu)D_s} \\ a_{31} & a_{32} & -1 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ M_e \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{q_{ua}(D+T_1)^3}{64\times(1+\mu)D_s} \\ 0 \end{bmatrix} \text{ --(19)}$$

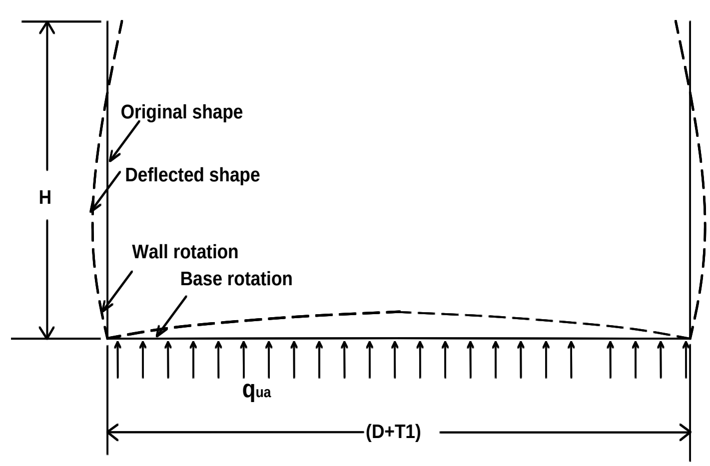

Fig. 2 The deflected shape of cylindrical tank at mean diameter for empty case.

Statement of The Problem

The aim of this paper is to obtain the tank optimum dimensions and the amount of reinforcements so that the cost of tank is minimum. During optimization process some parameters are considered as constants while the others are as the design variables. The constant parameters should be given at the start of optimization operation, these include the required design volume of tank ($Vreq$), the adequate free board ($F.B$), the bearing capacity of soil (q_{all}), specification for both concrete and steel reinforcement and unit cost for concrete, steel, and framework.

The Design Variables

The design variables are the geometric dimensions and the different steel reinforcement areas. The geometric dimensions include:

1- H : Height of water without adequate free board;

2- D : Inner diameter of tank;

3- T_1 : Thickness of cylindrical wall;

4- T_2 : Thickness of circular base;

In addition, the types of steel reinforcement include:

1- Ash : total area of horizontal tension steel reinforcement in cylindrical wall (two layers, one in each face);

2- $As1$: area of continuous vertical steel reinforcement at outer face of cylindrical wall per unit length;

3- $As2$: area of continuous vertical steel reinforcement at inner face of cylindrical wall per unit length;

4- $Asvp$: area of curtailed vertical steel reinforcement at outer face of cylindrical wall per unit length;

5- $Asvn$: area of curtailed vertical steel reinforcement at inner face of cylindrical wall per unit length;

6- Ast : area of steel reinforcement at top of base per unit length;

7- Asb : area of steel reinforcement at bottom of base per unit length.

The details of these design variables are illustrated in Fig. 3 where L1 and L2 are lengths of $Asvp$ and $Asvn$ measured from the top of the base; and:

$L_p = \beta \times (D + T_1)^2$ and

$$\beta = \sqrt{\frac{1}{4} - \frac{16M_e}{q_{ua}(3+\mu)(D+T_1)^2}} \text{ ------------------(20)}$$

The Objective Function

The objective function is defined as the total cost of tank (material and labor) for both the wall and the base. This includes the followings:

1-Cost of concrete including cost of materials, mixing, placing and curing.

2-Cost of various steel reinforcement in tank. This cost includes the material and labor costs.

3-Cost of formwork.

The cost of the wall is obtained by multiplying the circumference of tank cross section {estimated on the basis of mean diameter as $\pi(D+T_1)$ } by the cost of unit length of the wall (Wuc). The details of wall components for a unit width are shown in Fig. (3.a). The cost of wall per unit length is given as:

$$Wuc = H_t \times T_1 \times Cc + \{Ash + (H_t + T_2)(As1 + As2) + Asvp \times (L1 + T_2) + Asvn \times (L2 + T_2)\}Cs + 2H_t \times Cf \quad (21)$$

Therefore, the cost of wall (WC) is given as:

$$WC = \pi(D+T_1) \times [H_t \times T_1 \times Cc + \{Ash + (H_t + T_2)(As1 + As2) + Asvp.(L1 + T_2) + Asvn.(L2 + T_2)\}Cs + 2H_t \times Cf] \text{ - - - - - } (22)$$

The cost of base (BC) is determined from the following equation:

$$BC = \pi \times (D/2 + T_1)^2 \times T_2 \times Cc + 2\pi \{(D/2 + T_1 + T_2/2)^2 \times (Ast + Asb) - (\beta \times (D + T_1)/2)^2 \times (Asb - A_{sb\min}/2)\} \times Cs + 2\pi \times \{(D/2 + T_1) \times T_2 \times Cf \quad (23)$$

Then the objective function F can be written after rearrangement as follows:

$$F = \pi\{(D + T_1)(H_t \times T_1) + (D/2 + T_1)^2 \times T_2\} \times Cc + \pi\{[Ash + (H_t + T_2)(As1 + As2) + Asvp(L1 + T_2) + Asvn(L2 + T_2)] \times (D + T_1) + 2(D/2 + T_1 + T_2/2)^2 \times (Ast + Asb) - 2 \times (\beta \times (D + T_1)/2)^2 \times (Asb - A_{sb\min}/2)\} \times Cs + 2\pi\{H_t \times (D + T_1) + (D/2 + T_1) \times T_2\} \times Cf \quad (24)$$

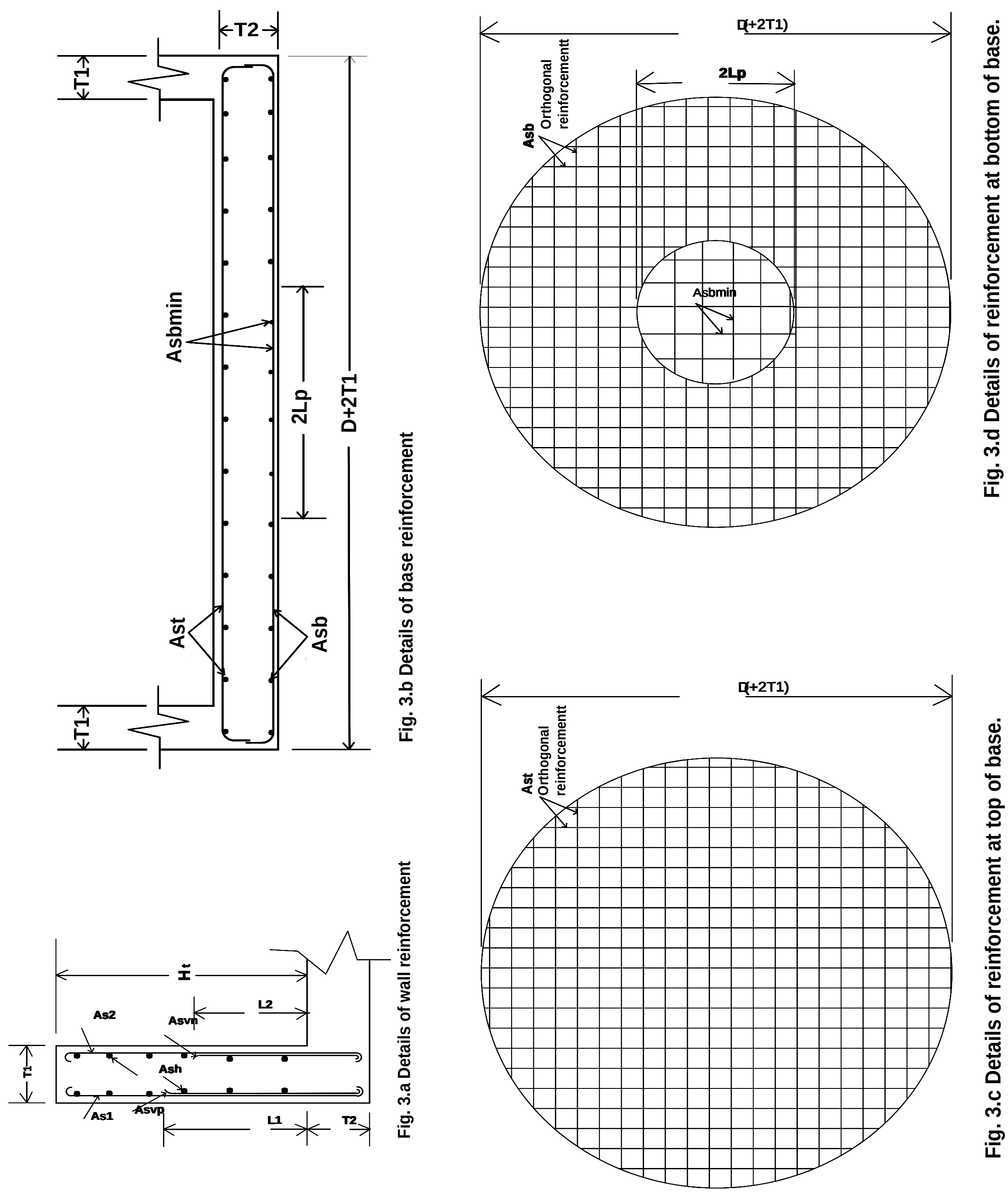

Fig. 3.a Details of wall reinforcement

Fig. 3.b Details of base reinforcement

Fig. 3.c Details of reinforcement at top of base.

Fig. 3.d Details of reinforcement at bottom of base.

Fig. 3 Details of design variable of the problem.

Design Constraints

1- General constraints

The first constraint ensures that the required design capacity is satisfied by available volume provided by design variables as shown below:

$Vreq - \pi D^2 H / 4 \leq 0$ ----------------------- (25)

The second constraint makes sure that the maximum tank pressure on ground (q_u) is less than allowable bearing capacity of soil (q_{all}), i.e.

$q_u - q_{all} \leq 0$ ---------------------------------- (26)

The third constraint ensures that all the design variables (except T_1 and T_2) must be positive value, i.e.

$-X_i \leq 0, \quad i = 1,2,..,n$ -------------------- (27.a)

For T_1 and T_2 the following constraints must be satisfied:

$T_{min} - T_i \leq 0, \quad i = 1,2$ ---------------------- (27.b)

where T_{min} : Minimum thickness specified by code requirement and equal to 200 mm.

2- Ultimate Limit State Constraints

For the flexural behavior constraints, the moments of resistance per unit length at the critical sections (M_u) should not be less than the bending moments per unit length due to the ultimate loads (M). These are represented by:

$M \leq M_u$ ------------------------------------ (28)

where

$M_u = (0.87 f_y) \times As \times z$ -------------------(29)

$z = (1 - \frac{1.1 f_y As}{f_{cu} bd}) \times d$ ---------------- (30)

b : is the width of the section;

d : is the effective depth of tension reinforcement;

As : is the area of tension reinforcement for bending resistance;

z: is the lever arm, which is not greater than 0.95d;

f_{cu} : is the characteristic cube strength of the concrete;

f_y : is the characteristic strength of the reinforcement.

This constraint is applied to both the wall and the base at junction joint and at the center of the base.

Another constraint is employed to specify that the tension reinforcement of the section (A_{si}) is not less than the minimum area ($A_{s\,min}$) required by the code, i.e.

$A_{s\,min} \leq A_{si}$ ------------------------------- (31)

where:

$A_{s\,min} = 0.003 \times b \times t$ for deformed bars ($f_y = 410$ MPa)

t : the thickness of the section. This constraint is applied to all sections of tank.

In addition, the maximum bar spacing must be less than the permissible value (S_{max}) given by the code, i.e.,

$S_i \leq S_{max}$ ---------------------------------- (32)

$S_{max} = 3 \times h$ where h is member thickness, this constraint is applied to the wall and to the base.

For shear behavior constraints, the section shear resistance V_u should be greater than the applied shear force V , i.e.:

$V \leq V_u$ -------------------------------------- (33)

where V is determined from equation (2) for wall and for base from the following equation [8]:

$V = q_{ua}(D + T_1)/4$ -------------------------- (34)

$V_u = v_c \times b \times d$ ---------------------------- (35)

v_c : the allowable shear stress of section according to BS 8110 . This constraint is applied to the wall and the base at junction joint.

For direct tension in the wall, the ultimate direct tension force T should be less than the section capacity in tension produced by steel reinforcement only T_u , i.e.

$T \leq T_u$ -------------------------------------- (36)

where

$T_u = Ash \times (0.87 f_y)$ ------------------------- (37)

3- Serviceability Limit State Constraints

These constraints ensure that the maximum crack widths w and steel stress f_s do not exceed the allowable permissible values (w_u , f_{su} , respectively), i.e.

$$w \le w_u \text{ ---------------------------------- (38)}$$

$$f_s \le f_{su} \text{ ---------------------------------- (39)}$$

where w_u = w_a = 0.1mm for inner faces of tank and = w_c = 0.3mm for outer faces of tank;

f_{su} = f_A =100 MPa for inner faces of tank and = f_{su} = 140 MPa for outer faces of tank.

This constraint is applied to the wall and the base at junction joint and at the center of the base.

Normalization of The Constraints

The above constraints should be normalized to give efficient coverage [11], therefore the normalized constraints become:

$$\frac{Vreq}{(\pi D^2 H/4)} - 1 \le 0 \text{ ------------------------- (40)}$$

$$\frac{q_u}{q_{all}} - 1 \le 0 \text{ ---------------------------------- (41)}$$

$$\frac{T_{\min}}{T_i} - T_i \le 0, \quad i = 1,2 \text{ --------------------- (42)}$$

$$\frac{M}{M_{ur}} - 1 \le 0 \text{ -------------------------------- (43)}$$

$$\frac{A_{s\min}}{A_{si}} - 1 \le 0 \text{ ------------------------------- (44)}$$

$$\frac{S_i}{S_{\max}} - 1 \le 0 \text{ ------------------------------- (45)}$$

$$\frac{V}{V_u} - 1 \le 0 \text{ --------------------------------- (46)}$$

$$\frac{T}{T_u} - 1 \le 0 \text{ --------------------------------- (47)}$$

$$\frac{w}{w_u} - 1 \le 0 \text{ --------------------------------- (48)}$$

$$\frac{f}{f_{su}} - 1 \le 0 \text{ --------------------------------- (49)}$$

Method of Solution

The present problem has been solved using interior penalty function method. There are several reasons for choosing interior penalty function method in solving the constrained optimization problem, one main reason is that the sequential nature of the method allows gradual approach to the criticality of the constraint, in addition, the sequential process permits a graded approximation to be used in the analysis of the system.

For resulted non-constrained problem, the computation of required derivatives of the objective function is very difficult, therefore the univariate algorithm, which is one of search methods, with quadratic interpolation technique for one-dimensional optimization has been adopted for the solution of this problem [11]. A computer program has been generated using Fortran language to get the required solution.

Results and Discussion

Table 1 shows the values of the basic parameters and Table 2 gives the optimal tank dimensions and the amount of steel reinforcements for design capacities equal to (100, 200, 300, 400, 500, 600) m^3. It has been observed from Table 2 that the optimum diameter and optimum height of tank simultaneously increase as the design capacity increases. The relation between optimum height and optimum diameter is given in Fig. (4). Also the design capacity of tank has no effect on the optimum wall thickness that is controlled by the minimum requirement specified by
increasing the design capacity [Table 2]. The distances L1 and L2 have a short length

compared with tank height. This result can be explained as the wall moments (positive or negative) have a maximum value at wall-base joint but after a short distance the moment values become so small that minimum steel areas are enough to resist these moments. The steel content (kg/m^3), which is defined as a ratio between the weight of reinforcement steel for entire tank in kilogram to the volume of concrete for entire tank in cubic meter, increases with increasing design capacity of tank. The relation between the total cost and the design capacity is shown in Fig. (5). It is approximately a linear relationship. In addition, from Table 2 the value of αH is less than 5 which indicate an important rule that the optimum tank type is a short tank where [12]:

$$\alpha H = \left(\frac{T_1}{I \times D^2}\right)^{0.25} H \qquad (50)$$

The effect of reducing bearing capacity is shown in Figs. (6) and (7) which reveal that reducing bearing capacity of soil causes a reduction in the optimal height because the main problem with bearing capacity of soil is to limit the applied pressure. Consequently the optimum height must be adjusted to satisfy the bearing capacity constraint when the applied pressure exceeds the allowable. This also causes the tank diameter to increase. In addition, it is observed that the total cost increases as the bearing capacity of soil decreases. A linear relationship is obtained between the design capacity and total cost.

The effect of variation of unit cost of concrete on optimum solution is depicted in Tables 2 and 3 and Fig. (8). It is clear that increasing concrete unit cost has highly influenced the optimum solution by increasing the optimum height and decreasing the optimum diameter. The base thickness behaves like the wall thickness, in that they are controlled by the minimum value specified by code. This effect may be attributed to that the optimum design moves towards decreasing concrete quantity in tank when concrete unit cost is increased. This effect causes higher value of steel content (kg/m^3).

From Table 4 and Fig. (9), it can also be seen that increasing steel unit cost affects the optimum height - optimum diameter relationship by increasing the optimum height and decreasing the optimum diameter. The base thickness is increases, the wall thickness remains unchanged, and the steel content (kg/m^3) is reduced with increasing steel unit cost. The above results can be attributed to that the optimum designs are varied in the direction of minimizing the steel quantity in the entire tank. Due to the increase of both concrete and steel unit cost the horizontal steel area in the wall is increased, because it is mainly proportional to the tank height.

Increasing formwork unit cost C_f has a considerable effect on the optimum height and optimum diameter and their relationship are illustrated in Fig. (10). Increasing C_f value causes an increase of tank diameter and a decrease of tank height.

Conclusions

It is found from the conducted study that:

1- The optimum tank is the one with walls of small height.

2-The total cost of tank is linearly increased with increase of the design capacity of tank.

3- Reducing bearing capacity of soil leads to increasing the diameter of tank and the total cost.

4- High concrete cost results in an increase of the tank height and steel content.

5- High steel cost results in an increase of the tank height and a decrease of steel content.

6- For high values of formwork cost, the tank height decreases.

Table 1 The values of the basic parameters used in the analysis.

Parameter	Value	Parameter	Value	Parameter	Value
B.C	**60 kN/ m²**	f_{ba}	**1.9 MPa**	f_A	**100 MPa**
F.B	**0.3 m**	w_a	**0.1 mm**	f_C	**140 MPa**
E	**200000 MPa**	w_c	**0.3 mm**	**Cc**	**100000 I.D/ m³**
f_{cu}	**25 MPa**	c.c (inner faces)	**40 mm**	**Cs**	**1000000 I.D/ ton**
f_y	**410 MPa**	c.c (outer faces)	**40 mm**	**Cf**	**10000 I.D/ m²**

c.c: minimum concrete cover specified by code BS 5337.

Table 2 Optimum design of tank for various tank capacity

Design Capacity of tank (m³)	100	200	300	400	500	600
Height (m)	2.50	3.21	3.70	3.95	4.27	4.44
αH	2.93	3.37	3.63	3.67	3.83	3.84
Diameter (m)	7.14	8.90	10.16	11.35	12.21	13.11
Wall Thickness (m)	0.200	0.200	0.200	0.200	0.200	0.200
Base Thickness (m)	0.200	0.200	0.207	0.210	0.216	0.221
Ash (mm²)	1680	2106	2400	2990	3807	4407
As1 (mm²)	302	320	324	305	301	311
As2 (mm²)	300	300	300	337	301	316
Asvp (mm²)	234	550	824	1075	1306	1488
Asvn (mm²)	0	19	146	201	353	440
Ast (mm²)	580	916	1163	1365	1547	1687
Asb (mm²)	537	866	1092	1283	1440	1558
L1 (m)	0.78	1.00	1.13	1.26	1.34	1.40
L2 (m)	0	0.49	0.59	0.60	0.68	0.71
Asvp / As1	0.78	1.72	2.54	3.52	4.34	4.79
Asvn / As2	0	0.06	0.49	0.60	1.17	1.39
Wall Cost (1000 I.D)	3254	5159	6805	8337	9894	11329
Base Cost (1000 I.D)	1695	3162	4660	6351	7922	9651
Wall Cost Ratio	0.66	0.62	0.59	0.57	0.56	0.54
Base Cost Ratio	0.34	0.38	0.41	0.43	0.44	0.46
Total Cost (1000 I.D)	4950	8320	11466	14688	17816	20980
Unit Cost (1000 I.D/ m³)	49.5	41.6	38.2	36.7	35.6	35.0
Steel content (kg/m³)	65	85	99	115	126	137

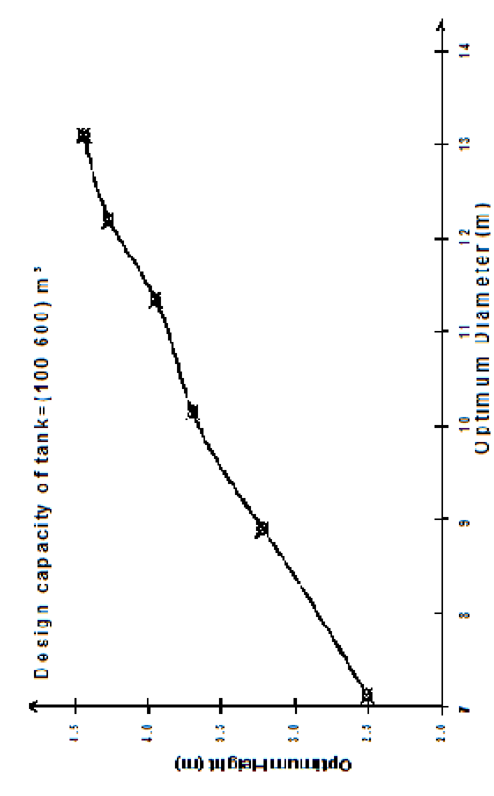

Fig. 4 The Relation between optimum diameter and optimum height

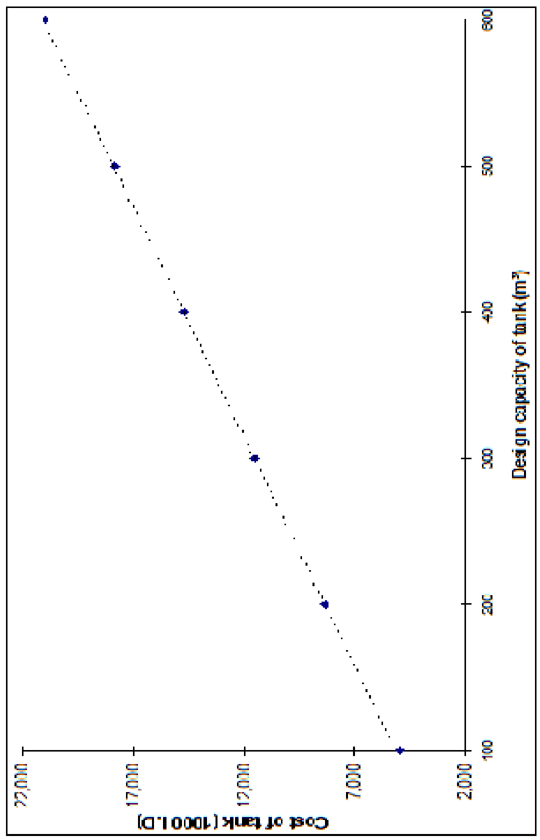

Fig. 5 The relation between design capacity of tank and total cost

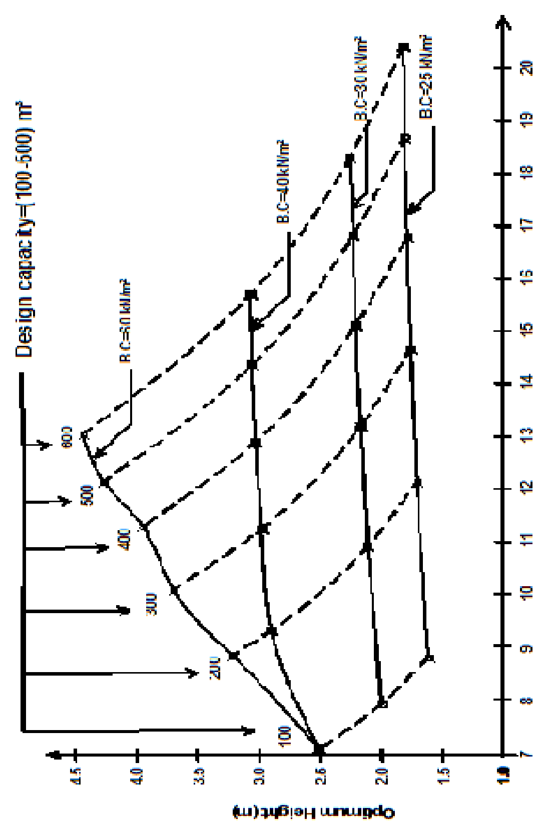

Fig. 6 The Relation between optimum diameter and optimum height for various bearing capacity values

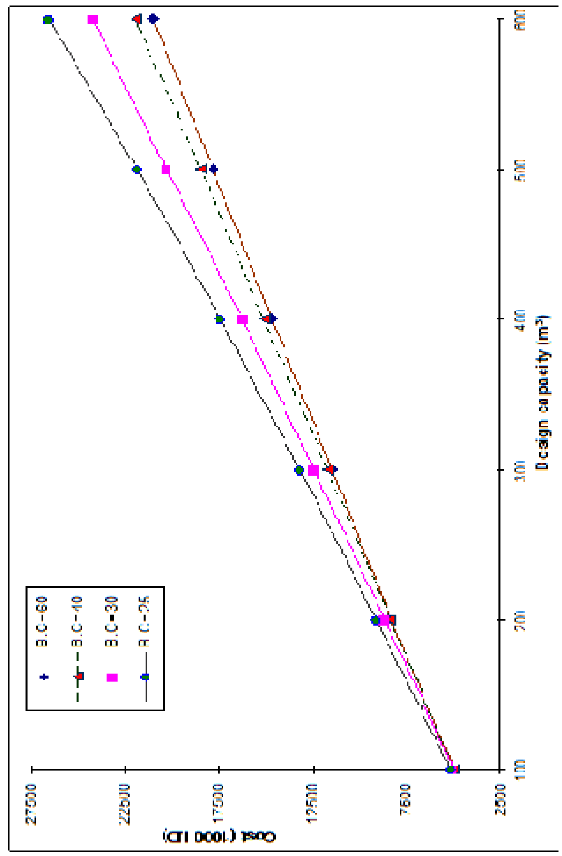

Fig. 7 The relation between design capacity and total cost for different bearing capacity values

Table 3 The optimum dimensions and concrete quantity for design capacities (100-600) m³ with different concrete unit cost values

Concrete Unit Price (1000 I.D/m³)	Design Capacity (m³)	Height (m)	Diameter (m)	Base Thickness (m)	Wall Thickness (m)	Concrete quantity (m³)	Steel/ Concrete (kg/m³)
250	100	2.75	6.8	0.200	0.200	21.58	66
	200	3.24	8.86	0.200	0.200	33.65	85
	300	3.83	9.98	0.200	0.200	43.39	103
	400	4.34	10.83	0.200	0.200	52.12	121
	500	4.83	11.48	0.200	0.200	60.28	137
	600	5.07	12.27	0.200	0.200	68.12	152
10,000	100	3.3	6.21	0.200	0.200	21.37	65
	200	4.11	7.87	0.200	0.200	33.12	85
	300	4.74	8.98	0.200	0.200	42.88	106
	400	5.12	9.97	0.200	0.201	51.56	128
	500	5.6	10.66	0.200	0.204	59.5	146
	600	5.9	11.38	0.200	0.206	66.91	162

Table 4 The optimum dimensions and reinforcement steel quantity for various design capacities (100-600) m³ with different steel unit cost values

Steel Unit Price (1000 I.D/ton)	Design Capacity (m³)	Height (m)	Diameter (m)	Wall Thickness (m)	Base Thickness (m)	Steel quantity (m³)	Steel/ Concrete (kg/m³)
1,000	100	3.35	6.16	0.200	0.202	0.18	65
	200	3.68	8.32	0.200	0.222	0.34	83
	300	4.24	9.49	0.200	0.246	0.5	84
	400	4.86	10.23	0.200	0.243	0.71	100
	500	5.2	11.06	0.200	0.257	0.91	108
	600	5.34	11.96	0.200	0.242	1.17	127
1,000,000	100	3.46	6.06	0.200	0.205	0.18	64
	200	3.9	8.08	0.200	0.224	0.34	76
	300	4.36	9.36	0.200	0.257	0.49	81
	400	4.92	10.17	0.200	0.280	0.66	89
	500	5.35	10.91	0.200	0.287	0.85	98
	600	5.51	11.77	0.200	0.288	1.06	108

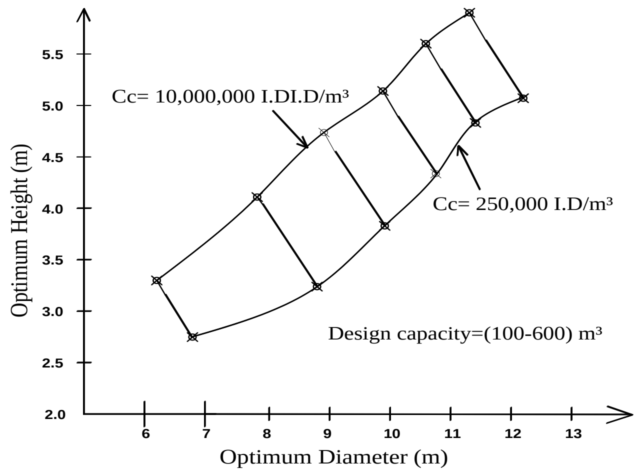

Fig. 8 The Relation between optimum diameter and optimum height for different concrete unit cost values

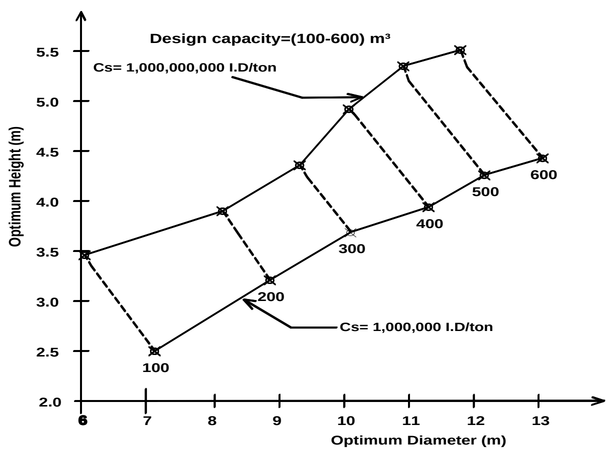

Fig. 9 The Relation between optimum diameter and optimum height for different reinforcement steel unit cost values

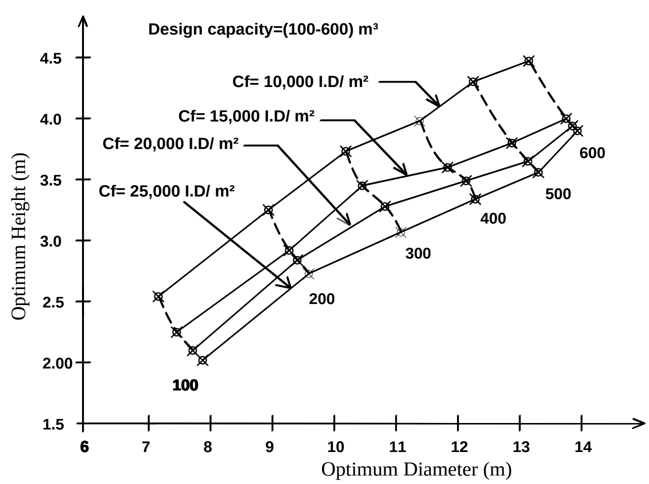

Fig. 10 The Relation between optimum diameter and optimum height for different formwork unit cost values

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